4 The new Keynesian model

4.1 Foundations: a classical monetary model (firms' block and equilibrium)

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Optimization by firms

The representative firm has the following production function:

$$Y_t = A_t N^{1-\alpha} \tag{1}$$

where A_t is the level of technology. $a_t = logA_t$ is an exogenous shock process. The firm maximizes profits in each period, taking P and W as given:

$$P_t Y_t - W_t N_t \tag{2}$$

Optimization by firms

Maximization given the production function yields:

$$\frac{W_t}{P_t} = (1 - \alpha) A_t N_t^{-\alpha} \tag{3}$$

This can also be written in terms of price equaling marginal cost:

$$P_t = \frac{W_t}{(1 - \alpha) A_t N_t^{-\alpha}} \tag{4}$$

Labour demand

One can write the previous equation in log-linear terms as:

$$w_t - p_t = a_t - \alpha n_t + log(1 - \alpha) \tag{5}$$

Equilibrium

Equilibrium implies goods market-clearing:

$$y_t = c_t$$

This condition is then combined with the optimality conditions for firms and households to derive the general equilibrium. We want to solve the model to obtain the relation of the variables with shocks and constants (functions of the deep parameters).

Equilibrium: output and employment

Note that:

$$w_t - p_t = a_t - \alpha n_t + log(1 - \alpha)$$

$$w_t - p_t = \sigma c_t + \varphi n_t$$

Implying:

$$a_t - \alpha n_t + log(1 - \alpha) = \sigma c_t + \varphi n_t \tag{6}$$

$$a_t - \alpha n_t + log(1 - \alpha) = \sigma y_t + \varphi n_t \tag{7}$$

Equilibrium: output and employment

Note that from the firms' production function:

$$y_t = a_t + (1 - \alpha)n_t \tag{8}$$

Which combined with:

$$a_t - \alpha n_t + log(1 - \alpha) = \sigma y_t + \varphi n_t \tag{9}$$

Ends in the equilibrium levels for output and employment which are a function of technology shocks:

$$n_t = \psi_{na} a_t + \nu_n \tag{10}$$

$$y_t = \psi_{ya} a_t + \nu_y \tag{11}$$

Equilibrium: real interest rate

The real interest rate $r_t = i_t - E_t\pi_{t+1}$ can be written in terms of output using:

$$c_t \simeq E_t c_{t+1} - \frac{1}{\sigma}(i_t - E_t\pi_{t+1} - \rho) \tag{12}$$

By the goods market clearing condition:

$$y_t \simeq E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t\pi_{t+1} - \rho) \tag{13}$$

We can thus rewrite the real rate as:

$$r_t = \rho + \sigma E_t \Delta y_{t+1} \tag{14}$$

$$r_t = \rho + \sigma\psi_{ya} E_t \Delta a_{t+1} \tag{15}$$

Equilibrium: real wage

The equilibrium real wage is given by:

$$w_t - p_t = a_t - \alpha n_t + log(1 - \alpha) = \psi_{wa} a_t + \nu_w \qquad (16)$$

Equilibrium: summary

$$n_t = \psi_{na} a_t + \nu_n$$

$$y_t = \psi_{ya} a_t + \nu_y$$

$$c_t = \psi_{ya} a_t + \nu_y$$

$$r_t = \rho + \sigma \psi_{ya} E_t \Delta a_{t+1}$$

$$w_t - p_t = \psi_{wa} a_t + \nu_w$$

Equilibrium: parameters

$$\psi_{na} = \frac{1-\sigma}{\sigma(1-\alpha)+\varphi+\alpha}$$

$$\psi_{ya} = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$$

$$\psi_{wa} = \frac{\sigma+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$$

$$\nu_n = \frac{log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$$

$$\nu_y = (1-\alpha)\nu_n$$

$$\nu_w = \frac{(\sigma(1-\alpha)+\varphi)log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$$